

FORMULARIO DE CÁLCULO I

FORMULAS DE ÁLGEBRA

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PRODUCTOS NOTABLES

$(x \pm y)^2 = x^2 \pm 2xy + y^2$	$(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$	FORMULA GENERAL $ax^2 + bx + c = 0 : x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x^2 - y^2 = (x - y)(x + y)$	$(x \pm a)(x \pm b) = x^2 \pm (a + b)x + ab$	Si $b^2 - 4ac > 0$ raíces reales distintas
$x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$	$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$	Si $b^2 - 4ac = 0$ raíces reales iguales Si $b^2 - 4ac < 0$ raíces complejas

EL BINOMIO DE NEWTON $(a + b)^n = a^n + \frac{n}{1!} a^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3}b^3 + \dots + b^n$

LOGARITMOS Definición: $\log_b N = x$ $b^x = N$ Donde: $N > 0$, $b > 0$ y $b \neq 1$ Propiedades:

$\log_b A + \log_b B = \log_b(AB)$	$\log_b A^n = n \log_b A$	$b^{\log_b A} = A$	$(\log_b a)(\log_a b) = 1$
$\log_b A - \log_b B = \log_b(A/B)$	$\log_b \sqrt[n]{A} = \frac{1}{n} \log_b A$	$\log_b A = \log_{(b^n)}(A^n)$	$\text{Colog}_b N = \log_b \left(\frac{1}{N}\right) = -\log_b N$
$\log_b b = 1 ; \log_b 1 = 0$	$\log_b A = (\log_a A) / (\log_a b)$	$\log_b A = \log_{(\sqrt[n]{b})}(\sqrt[n]{A})$	$\text{Antilog}_b x = b^x$

FORMULAS DE TRIGONOMETRÍA

$\begin{aligned} \text{sen}(-a) &= -\text{sen}a \\ \cos(-a) &= \cos a \\ \tan(-a) &= -\tan a \\ \tan \alpha &= \frac{\sin \alpha}{\cos \alpha} \\ \cot \alpha &= \frac{\cos \alpha}{\sin \alpha} \\ \sec \alpha &= \frac{1}{\cos \alpha} \\ \csc \alpha &= \frac{1}{\sin \alpha} \\ \cos^2 \alpha + \sin^2 \alpha &= 1 \\ \tan^2 \alpha + 1 &= \sec^2 \alpha \\ \cot^2 \alpha + 1 &= \csc^2 \alpha \\ \sin^2 a &= (1 - \cos 2a)/2 \\ \cos^2 a &= (1 + \cos 2a)/2 \\ \sin(2a) &= 2 \sin a \cos a \\ \cos(2a) &= \cos^2 a - \sin^2 a \end{aligned}$	$\begin{aligned} \sin(3x) &= 3 \sin x - 4 \sin^3 x \\ \cos(3x) &= 4 \cos^3 x - 3 \cos x \\ \sin(a \pm b) &= \sin a \cos b \pm \sin b \cos a \\ \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\ \tan(a \pm b) &= (\tan a \pm \tan b) / (1 \mp \tan a \tan b) \\ \sin a \cos b &= (\sin(a+b) + \sin(a-b))/2 \\ \cos a \cos b &= (\cos(a+b) + \cos(a-b))/2 \\ \sin a \sin b &= (\cos(a-b) - \cos(a+b))/2 \\ \sin a + \sin b &= 2 \sin \frac{(a+b)}{2} \cos \frac{(a-b)}{2} \\ \sin a - \sin b &= 2 \cos \frac{(a+b)}{2} \sin \frac{(a-b)}{2} \\ \cos a + \cos b &= 2 \cos \frac{(a+b)}{2} \cos \frac{(a-b)}{2} \\ \cos a - \cos b &= -2 \sin \frac{(a+b)}{2} \sin \frac{(a-b)}{2} \end{aligned}$	$\sin \frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$	DEG $\frac{\text{RAD}}{180^\circ} = \frac{\text{GRA}}{\pi} = \frac{\text{GRA}}{200^\circ}$
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α	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°	$\sinh x = (e^x - e^{-x})/2$	$\pi \approx 3.14$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π	$\cosh x = (e^x + e^{-x})/2$	$\frac{\pi}{2} \approx 1.57$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	$\tanh x = (e^x - e^{-x})/(e^x + e^{-x})$	$\sqrt{\pi} \approx 1.77$
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1	$\coth^2 x - \operatorname{senh}^2 x = 1$	$e \approx 2.72$
$\tan \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$-\infty$	0	$\operatorname{sech}^2 x = 1 - \tanh^2 x$	$e^2 \approx 7.38$
$\csc \alpha$	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	-1	∞	$\operatorname{csch}^2 x = \coth^2 x - 1$	$\sqrt{2} \approx 1.41$
$\sec \alpha$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	∞	1	$\operatorname{arcsen}h x = \ln(x + \sqrt{x^2 + 1})$	$\frac{\sqrt{2}}{2} \approx 0.70$
$\cot \alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$-\infty$	0	∞	$\operatorname{arccosh} x = \ln(x + \sqrt{x^2 - 1})$	$\sqrt{3} \approx 1.73$
												$\operatorname{arctanh} x = \frac{1}{2} \ln \frac{1+x}{1-x}$	$\frac{1}{\sqrt{3}} \approx 0.57$
												$\operatorname{arcsech} x = \ln[(1 + \sqrt{1 - x^2})/x]$	$\frac{\sqrt{3}}{2} \approx 0.86$

FORMULAS DE GEOMETRÍA

	Triángulo	Cuadrado	Rectángulo	Círculo	Elipse	Paralelogramo	Trapecio	Sector circular
Área	$\frac{1}{2}bh$	a^2	ab	πr^2	πab	bh	$\frac{(a+b)}{2}h$	$\pi r^2 \frac{\theta}{360}$
Perímetro		$4a$	$2a + 2b$	$2\pi r$				
	Cubo	Paralelepípedo	Tetraedro	Cilindro	Cono	Pirámide	Esfera	Elipsoide
Área	$6a^2$	$2(ab + ac + bc)$	$\sqrt{3}a^2$	$2\pi rh + 2\pi r^2$	$\pi rg + \pi r^2$		$4\pi r^2$	
Volumen	a^3	abc	$\frac{1}{12}\sqrt{2}a^3$	$\pi r^2 h$	$\frac{1}{3}\pi r^2 h$	$\frac{1}{3}A_b h$	$\frac{4}{3}\pi r^3$	$\frac{4}{3}\pi abc$

INECUACIONES

$(a, b) = \{x \in \mathbb{R} : a < x < b\}$	$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$	Si $a < b$ y $c < 0 \rightarrow ac > cb$	
$ a = \begin{cases} a & \text{si } a \geq 0 \\ -a & \text{si } a < 0 \end{cases}$	Si: $x, b \in \mathbb{R}, b > 0$ Entonces $ x \leq b \Leftrightarrow -b < x < b$	Si: $x, b \in \mathbb{R}, b > 0$ Entonces $ x \geq b \Leftrightarrow x \leq -b \vee x \geq b$	Si: $a, b \in \mathbb{R}$ Entonces $ ab = a b $
Si: $a, b \in \mathbb{R}, b \neq 0$ Entonces $ a/b = a / b $	Si: $a, b \in \mathbb{R}$ Entonces $ a + b \leq a + b $	Si: $a, b \in \mathbb{R}$ Entonces $ a - b = b - a $	$ x^n = x ^n$

OPERACIONES a es positivo

$\frac{0}{a} = 0$	$\frac{0}{\infty} = 0$	$\frac{\infty}{a} = \infty$	$a^0 = 1$	$\infty^\infty = \infty$	$0^\infty = 0$	$\infty^a = \infty$
$\frac{a}{0} = \infty$	$\frac{\infty}{0} = \infty$	$\frac{a}{\infty} = 0$	$a + \infty = \infty$	$\infty \infty = \infty$	$a^\infty = \infty$ si $a > 1$	$\log 0 = -\infty$
$a \infty = \infty$	$0 \infty = \infty$	$a \infty = \infty$	$\infty + \infty = \infty$	$\infty + \infty = \infty$	$a^\infty = 0$ si $a < 1$	$\log \infty = \infty$
INDETERMINACIONES	$\frac{0}{0}$	$\frac{\infty}{\infty}$	$\infty - \infty$	$0 \cdot \infty$	∞^0	1^∞
LÍMITES COMUNES	$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$	$\lim_{x \rightarrow 0} (1+x)^{1/x} = e$	$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$	$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$	$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$

DERIVADAS Definición: $F'(x) = \frac{dF}{dx} = y' = \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{F(x+h)-F(x)}{h}$; $h = \Delta x$ Es el incremento de 'x'

$(a') = 0$	$(\operatorname{sen} x)' = \cos x$	$(u^v)' = u^v (v' \ln u + u' v/u)$	$(x^x)' = x^x (\ln x + 1)$	$(\operatorname{arcsen} x)' = \frac{1}{\sqrt{1+x^2}}$
$(ax)' = a$	$(\cos x)' = -\operatorname{sen} x$	$(\operatorname{arcse} n x)' = 1/\sqrt{1-x^2}$	$(x)' = x /x, x \neq 0$	$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$
$(x^m)' = mx^{m-1}$	$(\tan x)' = \sec^2 x$	$(\operatorname{arccos} x)' = -1/\sqrt{1-x^2}$	$(\operatorname{senh} x)' = \cosh x$	$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$
$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(\cot x)' = -\csc^2 x$	$(\operatorname{arctan} x)' = 1/(x^2+1)$	$(\operatorname{cosh} x)' = \operatorname{senh} x$	$(\operatorname{arccoth} x)' = \frac{1}{1-x^2}$
$(a^x)' = a^x \ln a$	$(\operatorname{sec} x)' = \operatorname{sec} x \tan x$	$(\operatorname{arccot} x)' = -1/(x^2+1)$	$(\operatorname{tanh} x)' = \operatorname{sech}^2 x$	$(\operatorname{arcsech} x)' = \frac{-1}{x\sqrt{1-x^2}}$
$(e^x)' = e^x$	$(\operatorname{csc} x)' = -\csc x \cot x$	$(\operatorname{arcsec} x)' = 1/(x\sqrt{x^2-1})$	$(\operatorname{coth} x)' = -\operatorname{csch}^2 x$	$(\operatorname{arcsech} x)' = \frac{-1}{x\sqrt{1+x^2}}$
$(\ln x)' = 1/x$	$(u v)' = u'v + u v'$	$(\operatorname{arccsc} x)' = -1/(x\sqrt{x^2-1})$	$(\operatorname{sech} x)' = -\operatorname{sech} x \operatorname{tanh} x$	$(\operatorname{arccsch} x)' = \frac{-1}{x\sqrt{1+x^2}}$
$(\log_b x)' = \frac{1}{x \ln b}$	$\left(\frac{u}{v}\right)' = \frac{u'v - u v'}{v^2}$	<td></td> <td></td>		
EC. RECTA (Punto – Pendiente)	Pendiente	Pendiente ortogonal	Ángulo	Ángulo entre pendientes
$y - y_0 = m(x - x_0)$	$m = f'(x_0)$	$m = -1/f'(x_0)$	$\theta = \arctan(m)$	$\varphi = \arctan\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

CRITERIOS PARA HALLAR LOS MÁXIMOS Y MÍNIMOS

Criterio de la 2da derivada	Criterio de la 1ra derivada	Criterio de comparación	Con dominios de crecimiento
$f''(x_0) > 0 \exists \text{mín.}$ $f''(x_0) < 0 \exists \text{máx.}$ $x_0 = \text{Es un punto crítico}$	Sea: $x_1 < x < x_2$ $f'(x_1) < 0 \text{ y } f'(x_2) > 0 \exists \text{mín}$ $f'(x_1) > 0 \text{ y } f'(x_2) < 0 \exists \text{máx}$ $x \text{ es el punto crítico}$ $x_1, x_2 \text{ son muy cercanos a } x$	Sea: $x_1 < x < x_2$ $f(x) < f(x_1) \text{ y } f(x) < f(x_2) \exists \text{mín}$ $f(x) > f(x_1) \text{ y } f(x) > f(x_2) \exists \text{máx}$ $x \text{ es el punto crítico}$ $x_1, x_2 \text{ tienen que ser muy cercanos a } x$	Si en el punto crítico pasa de decreciente a creciente $\exists \text{mín}$ Si en el punto crítico pasa de creciente a decreciente $\exists \text{máx}$ (Siempre que en el punto crítico \nexists un punto de inflexión y \nexists asíntota vertical)

INTEGRALES

AREA A = $\int_a^b [F(x) - g(x)] dx = \int_c^d [F(y) - g(y)] dy$ F(x) Es la curva superior g(x) Es la curva inferior F(y) Es la curva derecha g(y) Es la curva izquierda	AREA DE SUPERFICIE DE REVOLUCIÓN $S_x = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2\pi \int_c^d y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dy$ $S_y = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2\pi \int_c^d x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dy$
VOLUMEN DE REVOLUCIÓN $V_x = \pi \int_a^b [F(x)]^2 - [g(x)]^2 dx = 2\pi \int_c^d y[F(y) - g(y)] dy$ $V_y = 2\pi \int_a^b x [F(x) - g(x)] dx = \pi \int_c^d [F(y)]^2 - [g(y)]^2 dy$	CENTRO GEOMÉTRICO $\bar{x} = \frac{\int_a^b x[F(x)-g(x)] dx}{\int_a^b [F(x)-g(x)] dx} = \frac{\frac{1}{2} \int_c^d [F(y)-g(y)]^2 dy}{\int_c^d [F(y)-g(y)] dy}$ $\bar{y} = \frac{\frac{1}{2} \int_a^b [F(x)-g(x)]^2 dx}{\int_a^b [F(x)-g(x)] dx} = \frac{\int_c^d y[F(y)-g(y)] dy}{\int_c^d [F(y)-g(y)] dy}$
LONGITUD DE ARCO $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_c^d \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dy = \int_m^n \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$	
Sustitución Trigonométrica $x = a \tan \theta ; \sqrt{x^2 + a^2} = a \sec \theta$	$x = a \sec \theta ; \sqrt{x^2 - a^2} = a \tan \theta$

En las siguientes integrales se debe añadir al final la constante +C (solo si se trabaja con integrales indefinidas)

$\int a dx = ax$ $\int x dx = \frac{x^2}{2}$ $\int x^n dx = \frac{x^{n+1}}{n+1}$ solo si: $n \neq -1$ $\int \frac{1}{x} dx = \ln x $ $\int e^x dx = e^x$ $\int a^x dx = a^x / \ln a$ $\int \ln x dx = x \ln x - x$ $\int \operatorname{sen} x dx = -\cos x$ $\int \cos x dx = \operatorname{sen} x$ $\int \tan x dx = -\ln \cos x $ $\int \cot x dx = \ln \operatorname{sen} x $ $\int \sec^2 x dx = \tan x$ $\int \csc^2 x dx = -\cot x$ $\int \sec x dx = \ln \sec x + \tan x $ $\int \csc x dx = \pm \ln \csc x \mp \cot x $ $\int \sec x \tan x dx = \sec x$ $\int \csc x \cot x dx = -\csc x$	$\int u dv = uv - \int v du$ (Integración por partes) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \operatorname{arctan} \frac{x}{a}$ $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right $ $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right $ $\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln x + \sqrt{x^2 + a^2} $ $\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsen \frac{x}{a}$ $\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln x + \sqrt{x^2 - a^2} $ $\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln x + \sqrt{x^2 \pm a^2} $ $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \operatorname{arcsen} \frac{x}{a}$ $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln x + \sqrt{x^2 - a^2} $ $\int \operatorname{arctan} x dx = x \operatorname{arctan} x - \frac{1}{2} \ln 1+x^2 $ $\int \operatorname{arcsen} x dx = x \operatorname{arcsen} x + \sqrt{1-x^2}$ $\int e^{ax} dx = e^{ax}/a$ $\int \operatorname{sen}(ax) dx = -(1/a)\cos(ax)$	$\int \cos(ax) dx = (1/a)\operatorname{sen}(ax)$ $\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$ $\int x^m e^{ax} dx = \frac{x^m}{a} e^{ax} - \frac{m}{a} \int x^{m-1} e^{ax} dx$ $\int x^m (\ln x)^n dx = \frac{x^{m+1} (\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx$ $\int x^m \ln x dx = x^{m+1} \left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right)$ $\int (\ln x)^m dx = x (\ln x)^m - m \int (\ln x)^{m-1} dx$ $\int e^{ax} \operatorname{sen} bx dx = \frac{e^{ax}}{a^2+b^2} (a \operatorname{sen} bx - b \operatorname{cos} bx)$ $\int e^{ax} \operatorname{cos} bx dx = \frac{e^{ax}}{a^2+b^2} (a \operatorname{cos} bx + b \operatorname{sen} bx)$ $\int x^m \operatorname{sen} bx dx = \frac{-x^m}{b} \operatorname{cos} bx + \frac{m}{b} \int x^{m-1} \operatorname{cos} bx dx$ $\int x^m \operatorname{cos} bx dx = \frac{x^m}{b} \operatorname{sen} bx - \frac{m}{b} \int x^{m-1} \operatorname{sen} bx dx$ $\int \operatorname{sen}^m x dx = \frac{-\operatorname{sen}^{m-1} x \operatorname{cos} x}{m} + \frac{m-1}{m} \int \operatorname{sen}^{m-2} x dx$ $\int \cos^m x dx = \frac{\cos^{m-1} x \operatorname{sen} x}{m} + \frac{m-1}{m} \int \cos^{m-2} x dx$ $\int \tan^m x dx = \frac{\tan^{m-1} x}{m-1} - \int \tan^{m-2} x dx$ $\int \sec^m x dx = \frac{\sec^{m-2} x \operatorname{tan} x}{m-1} + \frac{m-2}{m-1} \int \sec^{m-2} x dx$
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